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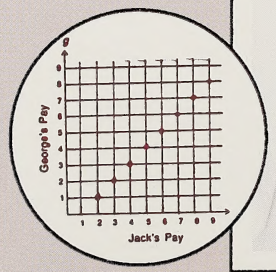
MODULE 4



MATHEMATICS 7

Distance Learning

Alberta EDUCATION

Mathematics 7

Module 4: Algebra

MODULE BOOKLET

Mathematics 7
Student Module
Module 4
Algebra
Alberta Distance Learning Centre
ISBN No. 0-7741-0135-0

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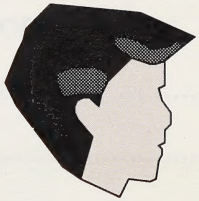
Welcome to Module 4!

We hope you'll enjoy your study of Algebra.

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,



turn on your audiocassette and listen.

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MODULE INTRODUCTION

What Lies Ahead

In this Module Introduction you will preview the Module components, and you will discover how the module will be evaluated.

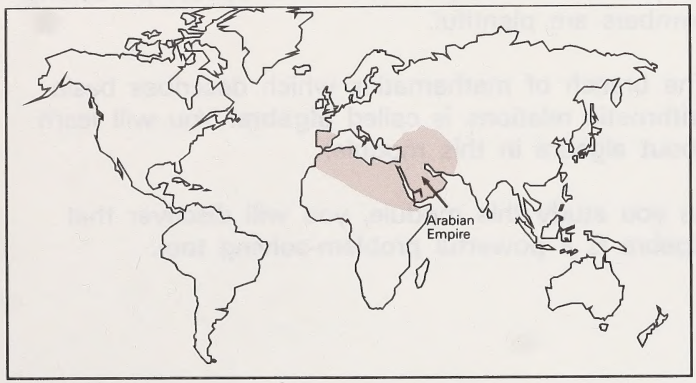
Working Together

As you are aware, patterns and relationships among numbers are plentiful.

The branch of mathematics which describes basic arithmetic relations is called **algebra**. You will learn about algebra in this module.

As you study this module, you will discover that algebra is a powerful problem-solving tool.

You may be interested to learn that algebra was studied in Baghdad around the eighth century. At that time the Arabian empire stretched from Spain to India, and Baghdad was a cultural and scientific centre.



The English word *algebra* was borrowed from the Arabian word *al-jabr* which was used in the title of a book written by the Arabian mathematician, Al-Khuwarizmi. (He lived about 800.)

Another important person in the development of algebra was Rene Descartes (1596 - 1650).

Descartes was a French mathematician who developed the process of graphing algebraic relations using order pairs of numbers and of describing these relations with equations.

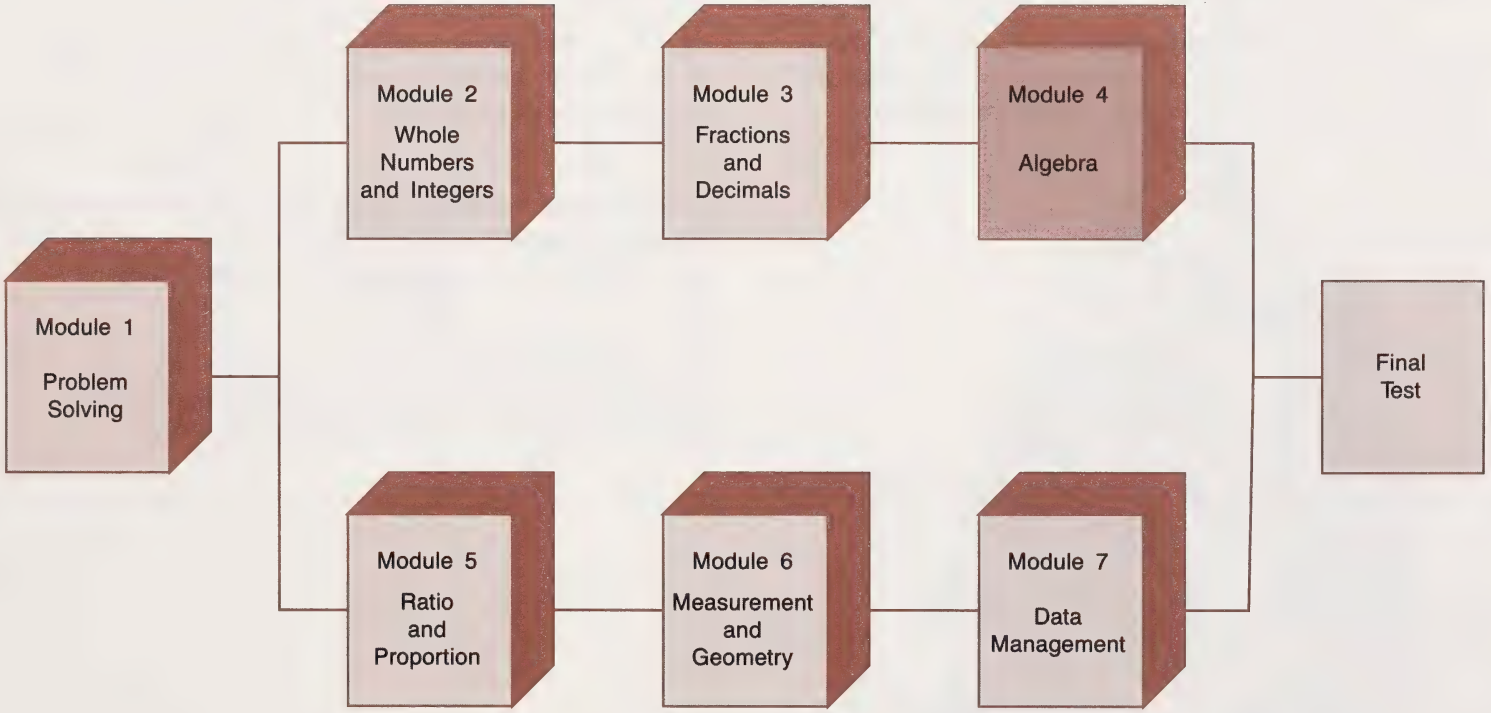
Here is an interesting story about how Descartes discovered coordinate geometry:

As a boy Descartes had poor health, so the rector of the Jesuit school he attended often allowed him to sleep in his room until he felt well enough to go to classes.

During those mornings in his room, Descartes often amused himself by watching a fly on the ceiling and by trying to form an equation that described its path.

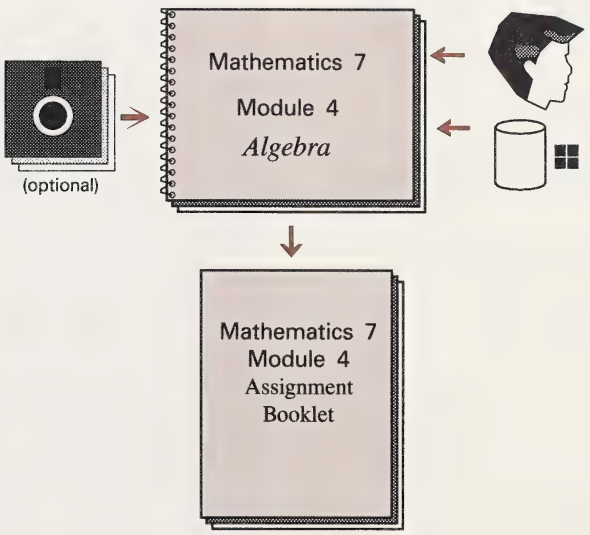
This practice led Descartes to develop the coordinate plane which is sometimes called the **Cartesian plane** in recognition of his work.

Course Overview



Mathematics 7 has seven modules and a final supervised test. This module booklet is part of Module 4.

Module 4 Components



This module booklet will give you instruction and practice in the skills and mathematical words you are required to learn. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional. There are print alternatives. You should see your learning facilitator to check your answers to the activities in this module booklet.

This module booklet is not to be submitted for a grade. **Your mark on this module will be determined by your work in the assignment booklet.**

Take time to preview this module booklet before beginning Section 1.

What Lies Ahead

This section will pretest these skills.

- using variables, expressions and equations to describe situations
- using learning aids to model and evaluate mathematical expressions
- evaluating expressions for given values of the variables by substitution
- using learning aids to demonstrate the concept of equality
- using learning aids to model and solve equations, and to verify the solution
- using guess-check procedures to solve equations
- plotting points given ordered pairs
- using tables, words, equations, number lines and graphs to describe a relation

Working Together

The pretest in this section will help you and your learning facilitator to discover your strengths and weaknesses in algebra.

Pretest

Space for Your Work

1. Translate each English phrase into a mathematical expression:

   a. four bottles plus six bottles

   b. six metres less than twice the width

2. Use learnings aids (cylinders, operation cards, and base 10 blocks) to do the following:

   a. Model $y + 4$ and evaluate if $y = 5$.

Space for Your Work

b. Model $3z - 1$ and evaluate if $z = 2$.

Space for Your Work

c. Model $2a^2 + 7$ and evaluate if $a = 3$.

Space for Your Work

3. Without using learning aids do the following

   a. Evaluate $3 + 4c$ if $c = 5$.

   b. Evaluate $2d - 1$ if $d = 1.2$.

   c. Evaluate $3 (s + t)$ if $s = 2$ and $t = 4$.

   d. Evaluate $3xy$ if $x = 0.1$ and $y = 0.4$.

Space for Your Work

4. What do the equal signs (=) signal in each of the following?

   a. $6 \times 2 - 1 = 11$

   b. $\frac{4}{5} = 0.8$

   c. $7 + 5 = 3 \times 4 - 1$

Space for Your Work

5. Are the following equations? Model each, and then answer "yes" or "no."

   a. $3 \times 2 = 8 - 1$

Space for Your Work

b. $5 + 4 + 3 = 3 \times 4$

6. Translate each English sentence into an equation:

   a. Five centimetres plus eight centimetres results in thirteen centimetres.

   b. Joseph's age minus four years is eight years.

Space for Your Work

7. Use learning aids (the cut-out scale in the appendix, operation cards, base 10 blocks and cylinders) to do the following:

   a. Model $y + 2 = 9$. Then solve the equation and verify the solution.

Space for Your Work

b. Model $5r + 3 = 13$. Then solve the equation and verify the solution.

Space for Your Work

8. Solve these equations and verify the solutions. Do not use learning aids.

   a. $a - 2 = 8$

   b. $3b = 27$

Space for Your Work

c. $2c + 9 = 17$

d. $4d + 1 = 2.16$

Space for Your Work

9. Plot the following

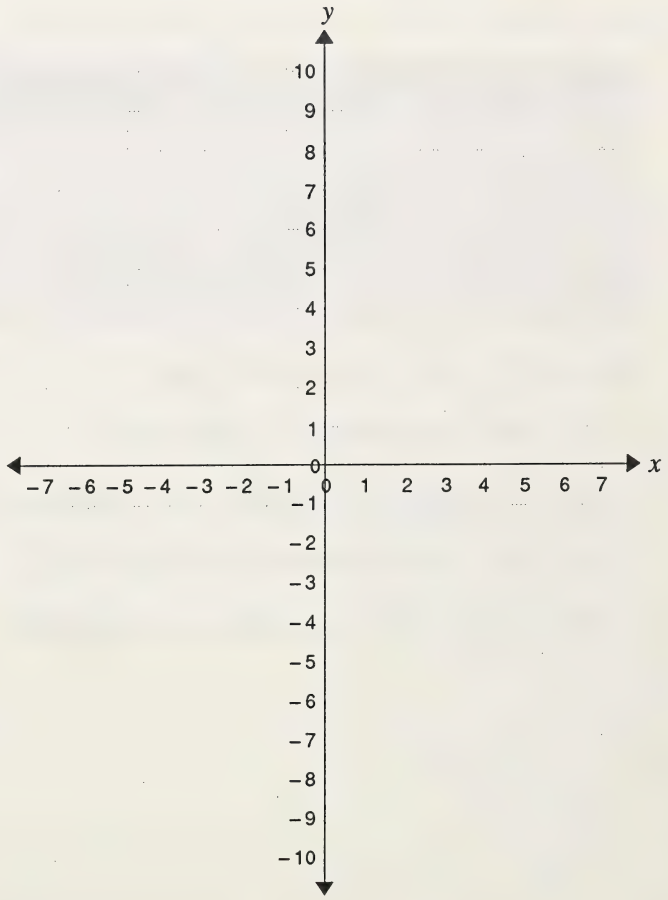
   a. (2, 5)

   b. (3, 7)

   c. (−2, 4)

   d. (−1, −5)

   e. (0, 3)



Space for Your Work

10. Jerrit unpacks cans in boxes at a grocery store.

Number of boxes (b)	Relation	Number of cans (c)
1	12 – 1	12
2	12 – 2	24
3	12 – 3	36
4	12 – 4	48
5	12 – 5	60

Describe the relationship several ways:

a. Use words to describe the relationship.

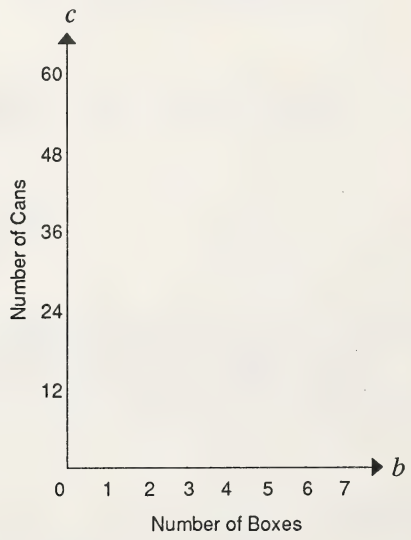
b. Write an equation to describe the relationship.

c. Use order pairs to describe the relationship.

d. Use number lines to describe the relationship.

Space for Your Work

e. Use a graph to describe the relationship.



See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will translate English words and phrases into mathematical expressions.

You will learn these words.

- variables
- mathematical expression
- algebraic expression

Working Together

What do each of these phrases mean?

- deux et trois
- two and three
- zwei und drei
- dos y tres

Different languages have been used, but all the phrases say the same thing. Each can be represented by this mathematical expression:

$$2 + 3$$

Mathematics is a common language understood by mathematicians all over the world.

Different English words and phrases can be translated into the same mathematical expression.

Examples

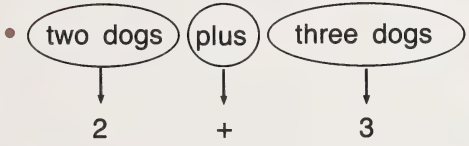
- add two and three
- the sum of two and three
- two plus three
- two increased by three

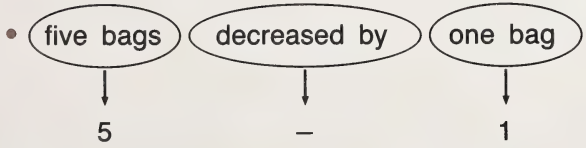
Each of the above phrases can be translated this way:

$$2 + 3$$

When you translate from English to mathematics, you sometimes can translate the words **in order**.

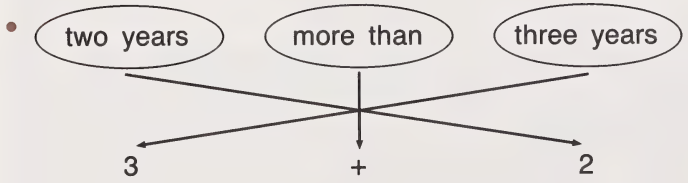
Examples

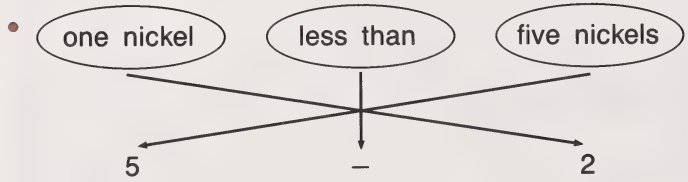




Sometimes when translating you need to change the order to reflect the meaning.

Examples





Note

Because of the commutative property for addition, $3 + 2 = 2 + 3$. So you can also translate "two more than three" as $2 + 3$.

However, the commutative property does not hold for subtraction, and $5 - 1 \neq 1 - 5$. So be careful when translating phrases involving subtraction.

Introductory Activities

Space for Your Work

Translate each English phrase into a mathematical expression.

1. seven baseballs increased by nine baseballs

2. the difference between nine poodles and six poodles

3. five groups of fifteen children

4. forty-eight pizzas divided by sixteen people

5. three years more than the difference of five years and two years

6. thirty-two tapes decreased by eight groups of three tapes

See your learning facilitator to check your answers and to receive further instructions.

A **variable** is a mathematical symbol that acts as a placeholder for a number that is not given or is not known.

Sometimes symbols are used as variables.

Examples

Usually, however, lower case (small) letters of the alphabet are used as variables.

Examples

a, m, x

Many English phrases can be translated by variables.

Examples

Phrase	Variable
Helen's mass	m
Ruth's wages	w
the length of the house	ℓ
Sam's age	a
Sue's height	h

An **algebraic expression** is an expression using variables. Algebraic expressions can be used to translate many English phrases.

Example

Phrase	Expression
Helen's mass decreased by two kilograms	$m - 2$
ten dollars more than Ruth's wages	$w + 10$
the length of the house divided by three	$\ell \div 3$ or $\frac{\ell}{3}$
five times Sam's age	$5a$
three more than twice Sue's height	$2h + 3$

Note

There are different ways to translate division.

$\ell \div 3$ or $\frac{\ell}{3}$

There are different ways to translate multiplication.

Method 1

When you translate "five times Sam's age," you usually do **not** use the multiplication sign ($\times$) between the number and the variable.

$$5a$$

If you do include the multiplication sign ($\times$), be careful it is not confused with the variable x.

$$5 \times a$$

↑ times

Method 2

When you translate three times six you can use a multiplication sign ($\times$) between the numbers.

$$3 \times 6$$

If you did not include the multiplication sign, you would have

$$36$$

This would be read as thirty-six and not three times six.

Method 3

Brackets can also be used to mean multiplication.

$$(3)(6) \qquad (5)(a)$$

Method 4

Occasionally raised dots are used to mean multiplication.

$$3 \cdot 6 \qquad 5 \cdot a$$

If you use the raised dot, be careful it is not mistaken as a decimal point.

Practice Activities

Space for Your Work

Computer Alternative

1. Do Lesson 3 on the Pre-Algebra disk of the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

2. Do Lesson 4 on the Pre-algebra disk of the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

Print Alternative

3. Translate the following situations into algebraic expressions.

   a. Marc's age increased by 2

   b. five times the distance from Muriel's house to school

   c. half the price of the car

   d. a number squared

   e. three less than the length

Space for Your Work

4. Translate each of the following situations into an algebraic expression:

   a. twice Omar's salary plus three dollars

   b. one less than two-thirds of a number

   c. four more than one-half of the distance

   d. the difference between eight and a number squared

   e. three more than twice Jason's mass

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Read the following phrases aloud.

   a. three times four, less two

   b. three times, four less two

   c. the sum of two, and five times three

   d. the sum of two and five, times three

2. How did you translate the commas when you read the phrases in question 1?

3. Mathematicians sometimes use parentheses to show order of operations. Translate each of the expressions in question 1 into a mathematical expression.

Space for Your Work

4. Translate each of the following into a mathematical or an algebraic expression:

   a. sixteen times, two plus a number

   b. sixteen times two, plus a number

   c. four times the length, plus eight centimetres

   d. four times, the length plus eight centimetres

5. Remember subtraction is sometimes translated in a different order. Translate the following:

   a. the difference of three, and two times a number

   b. the difference of three and two, times a number

   c. three less than four, times a number

   d. three less than, four times a number

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will use learning aids to model algebraic expressions, and to evaluate algebraic expressions.

In this section you will learn this word.

- evaluate

Working Together

In Modules 2 and 3 you used learning aids to model mathematical expressions such as these:

$$4 \times 3$$

$$(+5) + (-3)$$

$$\frac{3}{4} - \frac{1}{2}$$

$$0.25 \div 0.05$$

You also used models to evaluate these expressions. (Remember that to evaluate means to find the value of the expression.)

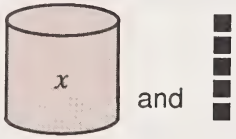
In this section you will use learning aids to model and to evaluate algebraic expressions.

You will use a cylinder to represent the number that is not specified. You will use base 10 blocks to represent the number that is given. You will use the operation cards in the appendix to represent $+$ or $-$.

Example 1

Evaluate $x + 5$ if $x = 2$.

Model the expression.



To evaluate the expression if $x = 2$, replace the cylinder with 2 units.

The result is 7.

So if $x = 2$, then $x + 5 = 7$.

Example 2

Evaluate $a - 3$ if $a = 7$.

Model the expression.

To evaluate the expression if $a = 7$, replace the cylinder with 7 units.

The result is 4.

So if $a = 7$, then $a - 3 = 4$.

Introductory Activities

Space for Your Work

Use cardboard cylinders from rolls of paper towel or rolls of toilet paper, or use cans to represent variables. Use the "and" and "take away" cards from the cut outs in the appendix to indicate addition and subtraction. Use the units from the base ten blocks to represent the numbers that are given.

1. a. Model $x + 5$.

b. Evaluate $x + 5$ if $x = 3$.

Space for Your Work

c. Evaluate $x + 5$ if $x = 10$.

2. a. Model $y - 2$.

Space for Your Work

b. Evaluate $y - 2$ if $y = 5$.

c. Evaluate $y - 2$ if $y = 4$.

See your learning facilitator to check your answers and to receive further instructions.

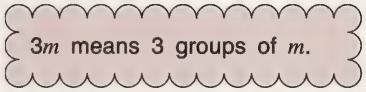
Working Together

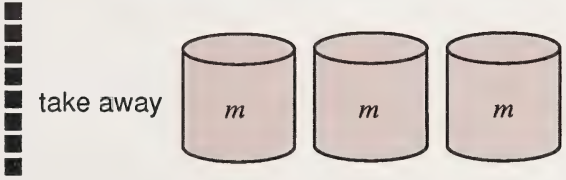
Now you will have a chance to model and evaluate more complex expressions.

Example 1

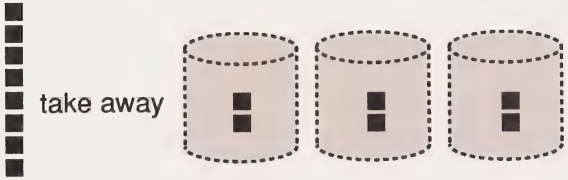
Evaluate $8 - 3m$ if $m = 2$.

First model the expression.





To evaluate the expression if $m = 2$, replace each cylinder with 2 units.



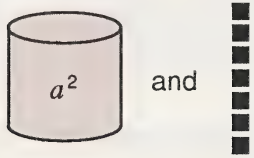
The result is 2.

So if $m = 2$, then $8 - 3m = 2$.

Example 2

Evaluate $a^2 + 7$ if $a = 3$.

First model the expression.



To evaluate the expression if $a = 3$, replace a^2 with 3^2, or 3 groups of 3 units.

The result is 16.

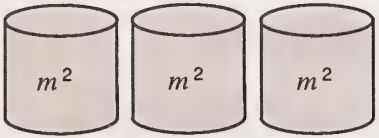
So if $a = 3$, then $a^2 + 7 = 16$.

Example 3

Evaluate $3m^2 - 1$ if $m = 2$.

Model the expression.

$3m^2$ means 3 groups of m^2.



take away ■

To evaluate the expression if $m = 2$, replace m^2 with 2^2, or 2 groups of 2 units.

take away ■

The result is 11.

So if $m = 2$, then $3m^2 - 1 = 11$.

Example 4

Evaluate $2(y + 1)$ if $y = 3$.

Model the expression.

$2(y + 1)$ means 2 groups of $y + 1$.

To evaluate the expression of $y = 3$, replace each y with 3 units.

The result is 8.

So if $y = 3$, then $2(y + 1) = 8$.

Practice Activities

Space for Your Work

Use cylinders to represent the variables. Use "and" and "take away" cards to indicate the operation. Use the units from base 10 blocks to represent the numbers that are given.

1. a. Model $3a - 2$.

b. Evaluate $3a - 2$ if $a = 5$.

Space for Your Work

c. Evaluate $3a - 2$ if $a = 2$.

2. a. Model $5 + 2b$.

b. Evaluate $5 + 2b$ if $b = 3$.

Space for Your Work

c. Evaluate $5 + 2b$ if $b = 0$.

3. a. Model $d^2 + 4$.

b. Evaluate $d^2 + 4$ if $d = 5$.

Space for Your Work

c. Evaluate $d^2 + 4$ if $d = 3$.

4. a. Model $2x^2 - 1$.

Space for Your Work

b. Evaluate $2x^2 - 1$ if $x = 4$.

c. Evaluate $2x^2 - 1$ if $x = 6$.

Space for Your Work

5. a. Model $2(3d - 1)$.

b. Evaluate $2(3d - 1)$ if $d = 5$.

Space for Your Work

c. Evaluate $2(3d - 1)$ if $d = 2$.

See your learning facilitator to check your answers and to receive further instructions.

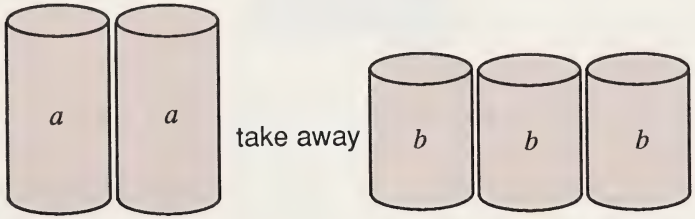
Working Together

Algebraic expressions may have more than one variable.

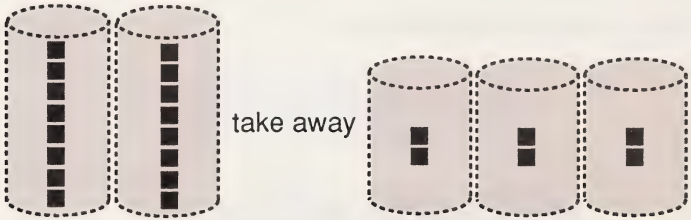
Example 1

Evaluate $2a - 3b$ if $a = 8$ and $b = 2$.

First model the expression.



To evaluate the expression if $a = 8$ and $b = 3$, replace the cylinders labeled a with 8 and the cylinders labeled b with 2.



The result is 10.

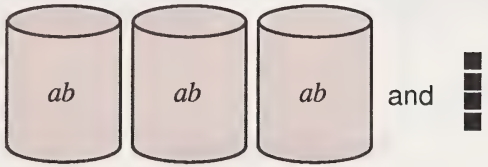
So if $a = 8$ and $b = 2$, then $2a - 3b = 10$.

Example 2

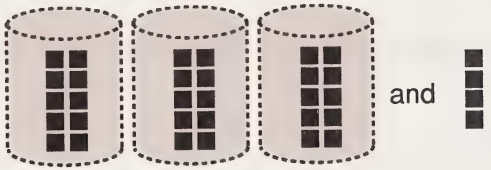
Evaluate $3ab + 4$ if $a = 2$ and $b = 5$.

First model the expressions.

$3ab$ means 3 groups of ab.



To evaluate the expression if $a = 2$ and $b = 5$, replace each cylinder labeled ab with 2×5, 2 groups of 5, or 10.



The result is 34.

So if $a = 2$ and $b = 5$, then $3ab + 4 = 34$.

Concluding Activities

Space for Your Work

Use cylinders to represent the variables. Use the "and" and "take away" cards to indicate the operation. Use the units from base 10 blocks to represent the numbers that are given.

1. a. Model $a + 3b$.

b. Evaluate $a + 3b$ if $a = 1$ and $b = 2$.

Space for Your Work

c. Evaluate $a + 3b$ if $a = 2$ and $b = 1$.

2. a. Model $2a + 3ab$.

Space for Your Work

b. Evaluate $2a + 3ab$ if $a = 2$ and $b = 5$.

c. Evaluate $2a + 3ab$ if $a = 5$ and $b = 2$.

Space for Your Work

3. a. Model $a^2 + 2ab + b^2$.

b. Evaluate $a^2 + 2ab + b^2$ if $a = 4$ and $b = 3$.

Space for Your Work

c. Evaluate $a^2 + 2ab + b^2$ if $a = 1$ and $b = 0$.

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will evaluate algebraic expressions without modeling them.

Working Together

In the previous section you used models to help you evaluate algebraic expressions.

You should now be ready to evaluate algebraic expressions using paper-and-pencil methods.

Evaluating Expressions With One Unknown

Example 1

Evaluate $8 + w$ if $w = 2$.

If $w = 2$, then $8 + w$

$= 8 + 2$

$= 10$

Example 2

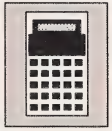
Evaluate $1.7 + g$ if $g = 0.5$.

If $g = 0.5$, then $1.7 + g$

$= 1.7 + 0.5$

$= 2.2$

Use can use a calculator if you wish.



Example 3

Evaluate $3a^2 + 5$ if $a = 7$.

If $a = 7$, then $3a^2 + 5$

$= 3 \times 7^2 + 5$

$= 3 \times 7 \times 7 + 5$

$= 3 \times 49 + 5$

$= 147 + 5$

$= 152$

Remember to follow the rules for order of operations (BEDMAS).

Example 4

Evaluate $2(32 - 5n)$ if $n = 4$.

If $n = 4$, then $2(32 - 5n)$

$= 2(32 - 5 \times 4)$

$= 2(32 - 20)$

$= 2 \times 12$

$= 24$

Remember to follow the rules for order of operations (BEDMAS).

Introductory Activities

Space for Your Work

Computer Alternative

1. Do Lesson 5 of the Pre-Algebra disk from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Remember if you need help press the SHIFT key and the ? key.

Print Alternative

2. a. Evaluate $n + 10$ if $n = 8$.

   b. Evaluate $2p$ if $p = 0.6$.

   c. Evaluate $5r - 2$ if $r = 7$.

Space for Your Work

d. Evaluate $3 + 4s$ if $s = 0.5$.

e. Evaluate $2n^2 + 1$ if $n = 4$.

f. Evaluate $0.5n - 0.1$ if $n = 2$.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Evaluating Expressions With More Than One Unknown

Example 1

Evaluate $5a + b$ if $a = 3$, and $b = 2$.

If $a = 3$ and $b = 2$, then $5a + b$

$= (5)(3) + 2$

$= 15 + 2$

$= 17$

Example 2

Evaluate $p^2 + 2pr + r^2$ if $p = 2$ and $r = 3$.

If $p = 2$ and $r = 3$, then $p^2 + 2pr + r^2$

$= 2^2 + (2)(2)(3) + 3^2$

$= 4 + 12 + 9$

$= 16 + 9$

$= 25$

Practice Activities

Space for Your Work

1. a. Evaluate $3a + 2b$ if $a = 2$ and $b = 7$.

b. Evaluate $p^2 - q^2$ if $p = 4$ and $q = 3$.

c. Evaluate $a^2 + 5ab - 6b^2$ if $a = 3$ and $b = 2$.

d. Evaluate $3(s + t)$ if $s = 0.2$ and $t = 0.5$.

Space for Your Work

2. Evaluate the following for $a = 4$, $b = 3$, $c = 2$.

   a. $a + (b + c)$ and $(a + b) + c$

   b. $a\,(b + c)$ and $ab + ac$

   c. $(a + b)(a - b)$ and $a^2 - b^2$

Space for Your Work

d. $\frac{ab + ac}{a}$ and $b + c$

e. $(a + b)(a + b)$ and $a^2 + 2ab + b^2$

3. What did you notice about question 2?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Sometimes you want to evaluate an algebraic for several values of the variable. Then a table is helpful.

Example

Evaluate $a + 5$ if $a = 1, 2, 3, 4, 5$ and 6.

Evaluate by substituting the first 3 values.

If $a = 1$, then $a + 5$

$= 1 + 5$

$= 6$

If $a = 2$, then $a + 5$

$= 2 + 5$

$= 7$

If $a = 3$, then $a + 5$

$= 3 + 5$

$= 8$

Then organize the values in a table to discover a pattern.

a	$a + 5$	Pattern
1	6	
2	7	+1
3	8	+1

Apply the pattern to find the last 3 values.

a	$a + 5$	Pattern
1	6	
2	7	+1
3	8	+1
4	9	+1
5	10	+1
6	11	+1

Concluding Activities

Space for Your Work

1. Complete the following tables. (Evaluate the first 3 values and use a pattern to help you find the last 3 values.)

a.

a	$a + 2$
1	
2	
3	
4	
5	
6	

b.

b	$b - 1$
1	
2	
3	
4	
5	
6	

Space for Your Work

c.

c	$3c$
1	
2	
3	
4	
5	
6	

d.

d	$3d + 2$
1	
2	
3	
4	
5	
6	

Space for Your Work

e.

f	$5f - 1$
1	
2	
3	
4	
5	
6	

2. Complete the following tables. (Evaluate the first 3 variables and use a pattern to help you find the last 3 values.)

a.

a	a^2
1	
2	
3	
4	
5	
6	

Space for Your Work

b.

b	$b^2 + 3$
1	
2	
3	
4	
5	
6	

c.

c	$c^2 - 1$
1	
2	
3	
4	
5	
6	

Space for Your Work

3. What did you notice about the patterns in questions 1 and 3?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will develop your understanding of these concepts.

- equality
- equations

Working Together

In the previous section the equal sign (=) was used as a symbol to signal that you are to find the value of an expression.

Example

3 + 2 − 1 =

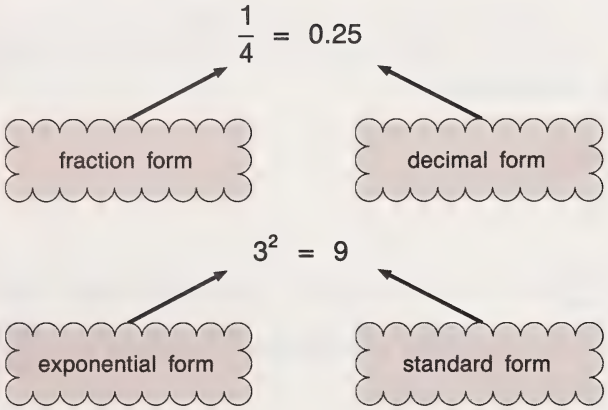
The [=] key on a calculator is used this way. After you have put in the expression, you press the [=] key to find the answer.

Key Press	Display
[3] [+] [2] [−] [1] [=]	4

In this section you will review other roles of the equal sign.

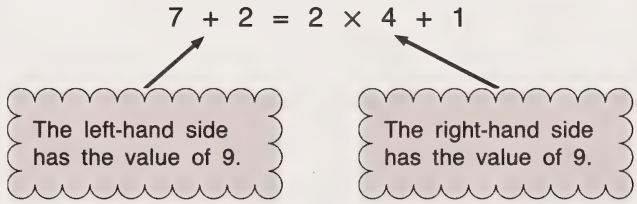
Sometimes the equal sign (=) is also used as a symbol to signal different forms of the same number.

Example



Sometimes the equal sign (=) is used to indicate that two expressions have the same value.

Example

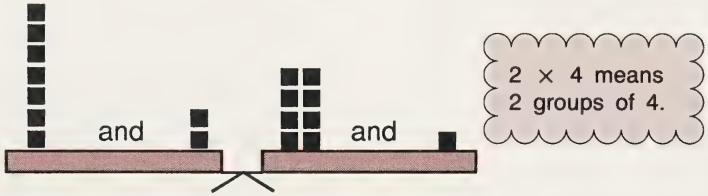


An **equation** is a mathematical sentence showing the left-hand side (LHS) and the right-hand side (RHS) have the same value.

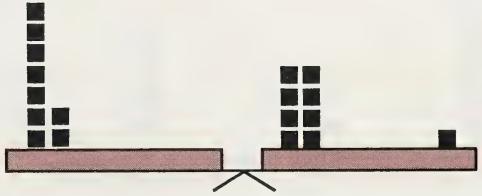
Example 1

Does $7 + 2 = 2 \times 4 + 1$?

You can make a model.



You can simplify each side.



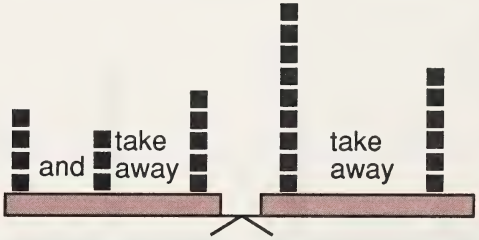
The value of each side is 9.

So LHS = RHS, and $7 + 2 = 2 \times 4 + 1$.

Example 2

Does $4 + 3 - 5 = 9 - 6$?

You can make a model.



You can simplify each side.

The left-hand side has a value of 2.
The right-hand side has a value of 3.

So LHS $\neq$ RHS, and $4 + 3 - 5 \neq 9 - 6$.

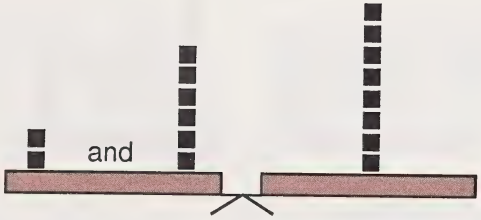
Note

$\neq$ means not equal to.

Equations are used to show how numbers are related.

Different equations can be written to express the relationship between 2, 6, and 8.

Example 1



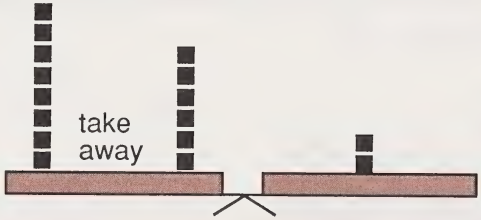
$2 + 6 = 8$

or

$8 = 2 + 6$

When writing an equation the left-hand-side (LHS) and the right-hand-side (RHS) may be reversed. The equality is still shown.

Example 2

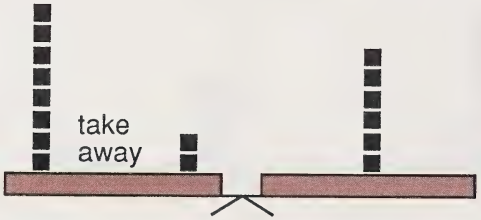


$8 - 6 = 2$

or

$2 = 8 - 6$

Example 3



$8 - 2 = 6$

or

$6 = 8 - 2$

Introductory Activities

Space for Your Work

1. What do the equal signs signal in each of the following?

   a. $5 \times 3 + 2 = 17$

   b. $\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

   c. $4 + 3 + 2 = 3 \times 3$

Space for Your Work

2. Model these equations using the cut-out equation scale and cut-out operation cards in the appendix, and the units from a set of base 10 blocks. Then perform the operations to verify that the scale is balanced.

   a. 2 + 3 + 5 = 10

Space for Your Work

b. $6 = 2 \times 3$

c. $10 - 4 = 1 + 2 + 3$

Space for Your Work

3. Are the following equations? Answer yes or no and explain why.

   a. $5 + 4 = 3 \times 3$

Space for Your Work

b. $9 - 1 = 1 + 2 + 3$

4. Write several equations to show the relationship between 3, 5, and 8.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Translating Sentences Into Equations

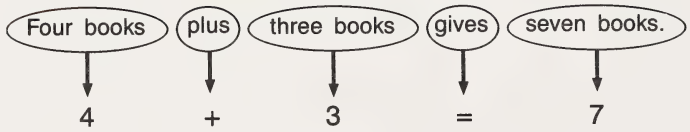
In Section 2 you translated English words and phrases into mathematical expressions.

Now you will translate English sentences into equations.

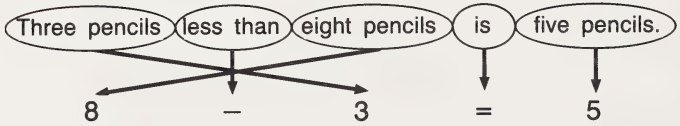
English words such as these can be translated by an equal sign (=).

- is
- is the same as
- results in
- gives the same result as
- gives
- equals

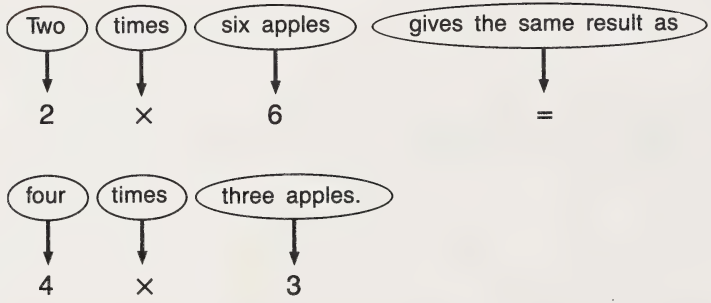
Example 1



Example 2



Example 3



Some word problems cannot be translated "word by word" into equations.

You must use your understanding of the relationship among the numbers to write an equation.

Example 1

Each student was given a notebook costing $0.80. It cost $24 to provide 30 students with a notebook.

Think

30 notebooks times $0.80 is $24.

Write an equation

$3 \times 0.80 = 24$

Example 2

An apartment owner has 100 m of parking space. She has 8 spaces for small cars and 10 spaces for large cars. A small car requires 5 m of space and a large car requires 6 m of space.

Think

8 times five metres plus 10 times 6 metres is 100 metres.

Write an equation

$8 \times 5 + 10 \times 6 = 100$

Practice Activities

Space for Your Work

Express each of the following as an equation.

1. Four oranges plus seven oranges results in eleven oranges.

2. Six shirts less than eight shirts is the same as two times one shirt.

3. You can buy five hamburgers with $4 if each burger costs $0.80.

4. You receive $3.02 from $5 if the purchase cost is $1.98.

5. Vlad's mass was 80 kg. He lost 10 kg and now his mass is 70 kg.

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Translate the following. Use brackets where needed.

   a. The square of nine, minus three, results in seventy-eight.

   b. The square of, nine minus three, gives thirty-six.

   c. The sum of eight and two, times six is sixty.

   d. The sum of eight, and two times six equals twenty.

Space for Your Work

2. Use brackets to make these statements true. The first one is done as an example.

   a. $39 \div 4 + 9 = 9 - 5 + 1$

   b. $27 + 9 \div 3 = 3 + 3 \times 2$

   c. $29 - 4 \times 3 + 8 = 200 + 25 \times 3$

   d. $17 - 18 \div 2 - 4 = 3 + 1 \times 3 - 8$

   e. $1 \times 84 \div 1 + 5 = 5 \times 2 + 4$

2. a. $39 \div (4 + 9) = 9 - (5 + 1)$

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

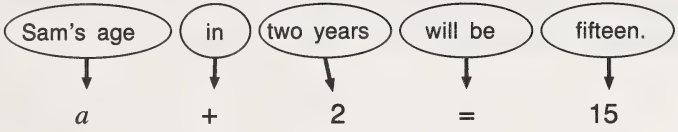
- translating English sentences into algebraic equations
- modeling algebraic equations
- solving algebraic equations using learning aids
- verifying solutions using learning aids

Working Together

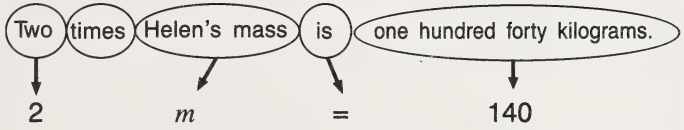
In the previous section you worked with numerical equations. In this section you will work with algebraic equations (equations that contain variables).

You can translate situations with unknowns into equations if you know how the variables and numbers are related.

Example 1



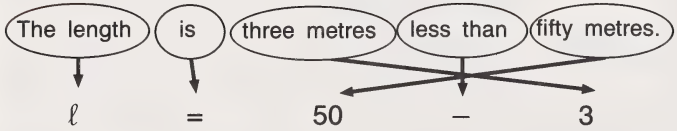
Example 2



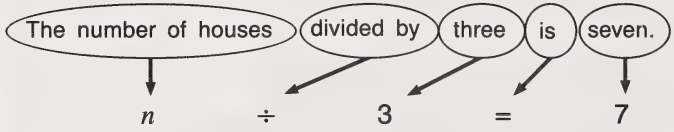
Note

Multiplication is understood in $2m$.

Example 3



Example 4



or

$$\frac{n}{3} = 7$$

Note

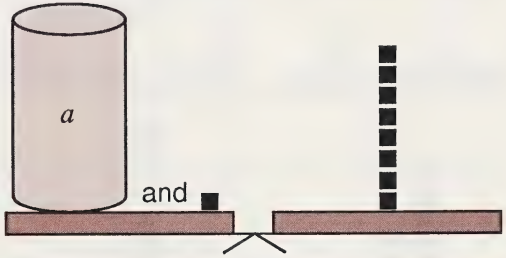
Division can be shown two ways.

Working Together

Algebraic equations are only true for certain values of the variable.

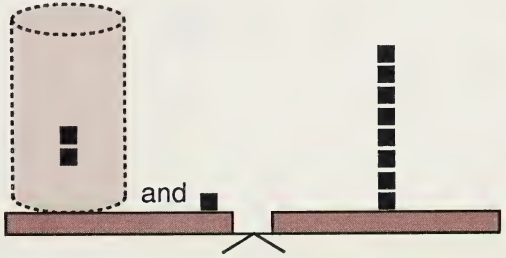
Example

Does $a + 1 = 8$?



Trial 1

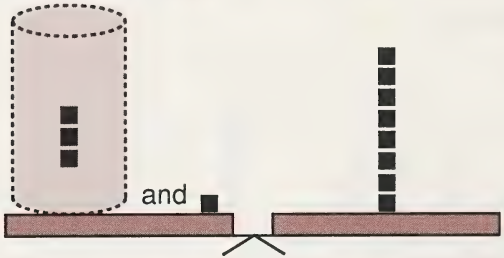
Try replacing a with 2.



If $a = 2$, the sides will not be balanced.

Trial 2

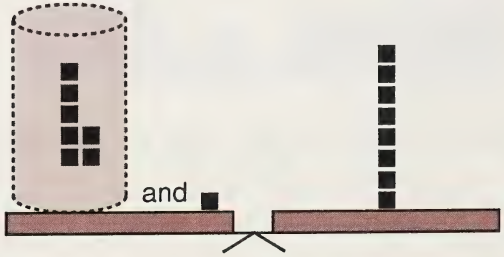
Try replacing a with 3.



If $a = 3$, the sides will not be balanced.

Trial 3

Try replacing a with 7.



If $a = 7$, the sides are balanced.

If $a = 7$, then $a + 1 = 8$.

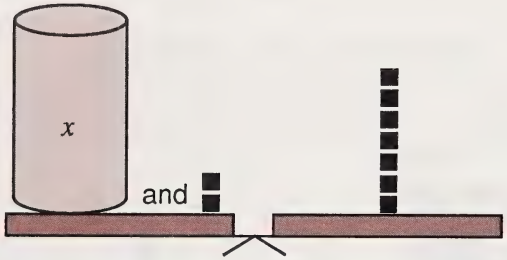
Algebraic equations are sometimes called **conditional equations** or **conditions**.

Learning aids can be used to solve equations. To **solve the equation** means to find the value of the variable (or the values of the variables) that make a true statement.

Example 1

Solve $x + 2 = 7$.

First model the equation.



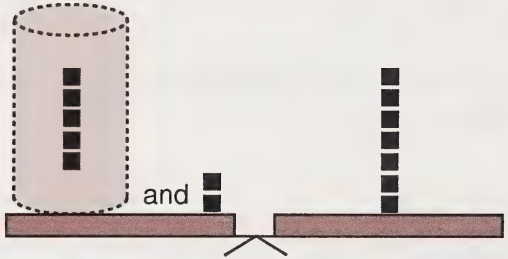
Try to think of value that will make the equation true.

What number plus 2 is 7?

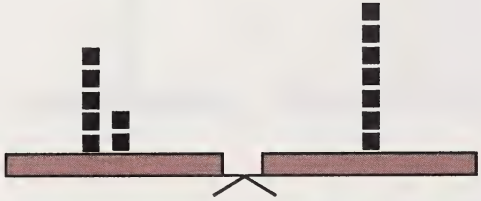
$\boxed{5} + 2 = 7$

So $x = 5$.

To verify the solution replace the cylinder with 5 units.



Simplify both sides.



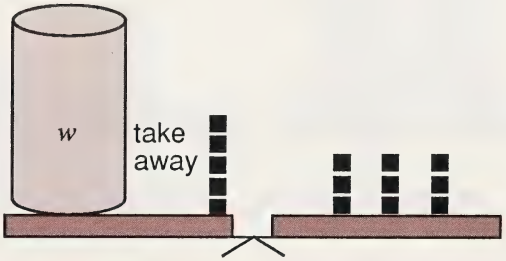
The result is 7 units on each side. The scale is balanced.

So $x = 5$.

Example 2

Solve $w - 5 = 9$.

First model the equation.



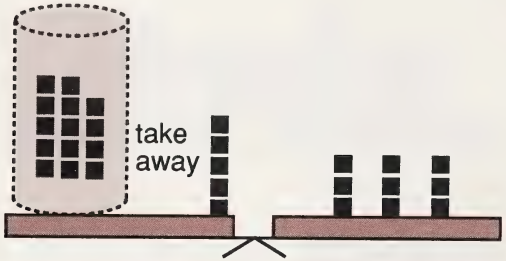
Then try to think of a value that will make the equation true.

What number take away 5 is 9?

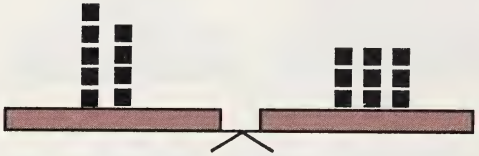
$$\boxed{14} - 5 = 9$$

So $w = 14$.

Next verify the solution, replace the cylinder with 14 blocks.



Finally simplify both sides.



The result is 9 on each side.

The scale is balanced.

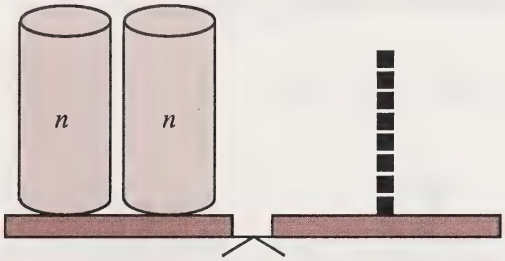
So $w = 14$.

Example 3

Solve $2n = 8$.

$2n$ means 2 groups of n.

First model the equation.



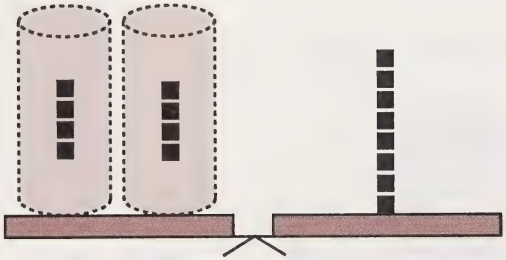
Then try to think of a number that makes the equation true.

Two times what number is 8?

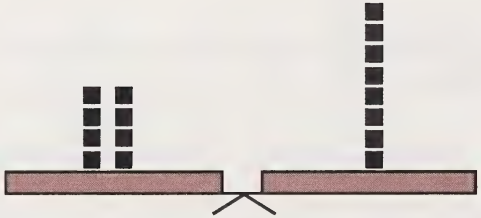
$$2 \times \boxed{4} = 8$$

So $n = 4$.

Next verify the solution by replacing each cylinder with 4.



Finally simplify both sides.



The result is 8 on each side.

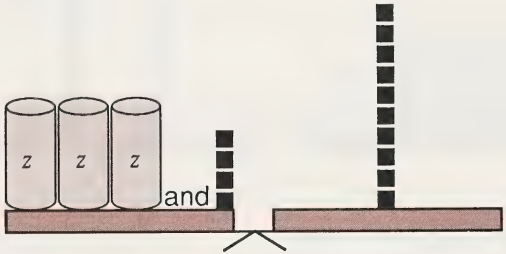
The scale is balanced.

So $n = 4$.

Example 4

Solve $3z + 4 = 10$.

First model the equation.



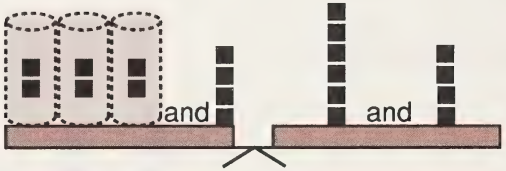
Then try to think of a number that makes the equation true.

3 times what number plus 4 is 10.

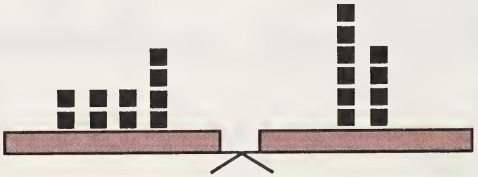
$$3 \times \boxed{2} + 4 = 10$$

So $z = 2$.

Next verify the solution by replacing each cylinder with 2.



Finally simplify each side.



The result is 10 on each side.

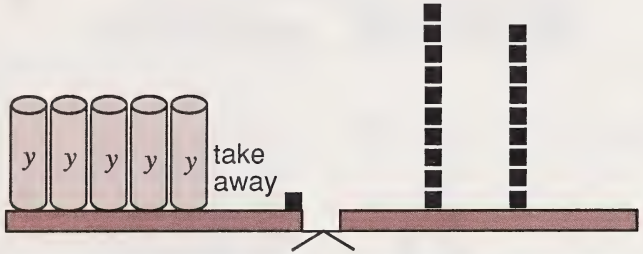
The scale is balanced.

So $z = 2$.

Example 5

Solve $5y - 1 = 19$.

First model the equation.



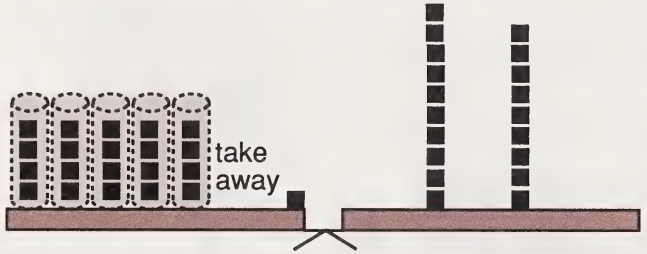
Then try to think of a number that makes the equation true.

Five times what number minus 1 is 19?

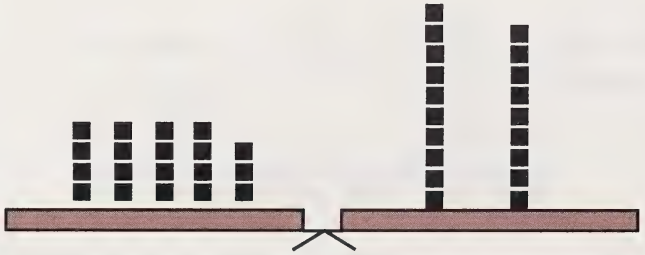
$$5 \times \boxed{4} - 1 = 19$$

So $y = 4$.

Next verify the solution by replacing each cylinder with 4.



Finally simplify each side.



The result is 19 on each side.

The scale is balanced.

So $y = 4$.

Practice Activities

Space for Your Work

Use the cut-out equation scale, operation cards, cylinders and base 10 blocks in the following.

1. a. Model $m + 2 = 7$.

b. Solve the equation. (Find the value that makes the equation true.)

Space for Your Work

c. Verify the solution. (Test to check it is true for this value of the variable.)

2. a. Model $w - 3 = 8$.

Space for Your Work

b. Solve the equation.

c. Verify the solution.

Space for Your Work

3. a. Model $4b = 8$.

b. Solve the equation.

c. Verify the solution.

Some word problems cannot be translated "word by word" into equations.

You must use your understanding of the relationship among the numbers to write an equation.

Example 1

André read 432 pages of his book. He had 58 pages left to read.

The number of pages in the book minus 432 is 58.

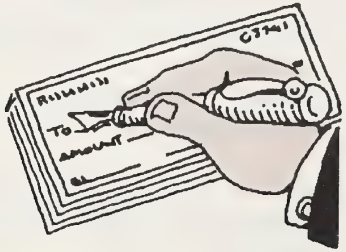
$$x - 432 = 58$$

Example 2

On Thursday Galen deposited in his account $40 at the bank. He then wrote a cheque for $25. Afterwards he had $354.98 in the account.

The amount of money in the account plus $40 minus $25 is $354.98.

$$m + 40 - 25 = 354.98$$



Introductory Activities

Space for Your Work

Print Alternative

1. Express each of the following as an equation.

   a. Alnoor's age minus two is seventeen.

   b. Three times Ruth's age gives eighteen.

   c. A number decreased by six is the same as four times eight.

   d. Jake's mass divided by nine is eight kilograms.

   e. Twelve less than four times the number of people in the room is fifty-six.

Computer Alternative

2. If you wish further practice translating a sentence into an equation, do Lesson 6 on the Pre-Algebra disk of *Computer Drill and Instruction: Mathematics, Level D* (SRA).

Space for Your Work

3. Express the following as equations.

   a. Marie has $85 in the bank at the end of December. This amount is $5 more than double the amount she had at the beginning of December.

   b. Soccer practices last for 2 hours and Andrew missed 3 practices. Andrew spent 54 hours in team practices.

Space for Your Work

c. The band has 12 people. If the choir gets one more person, there will be twice as many people in the choir as in the band.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

4. a. Model $3x + 5 = 38$.

b. Solve the equation.

Space for Your Work

c. Verify the solution.

5. a. Model $4p - 2 = 50$.

Space for Your Work

b. Solve the equation.

c. Verify the solution.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Sometimes equations have more than one variable.

Example 1

The number of pupils is 20 times the number of teachers.

$p = 20t$

Example 2

The number of students in the school is equal to the number of boys plus the number of girls.

$s = b + g$

Example 3

The number of sparrows at the bird feeder is 3 more than the number of bluejays.

$s = 3 + b$

Example 4

The length is two more than twice the width.

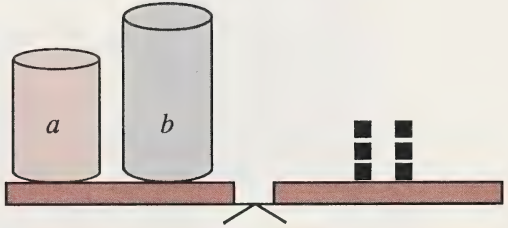
$\ell = 2w + 2$

How do you solve an equation with more than one variable?

Example

Solve $a + b = 6$.

First model the equation.



Then try to think of values that will make the equation true.

What number plus what other number is 6?

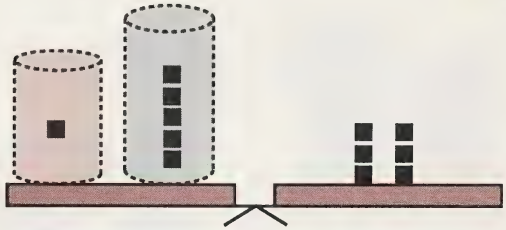
$$1 + 5 = 6$$
$$2 + 4 = 6$$

So $a = 1$ and $b = 5$

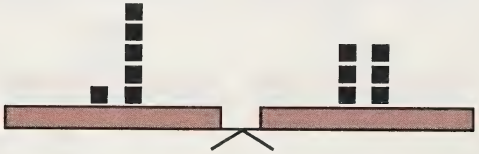
or

$a = 2$ and $b = 4$.

Next verify the first solution replacing the cylinder labelled a with 1 and the cylinder labelled b with 5.



Now simplify both sides.

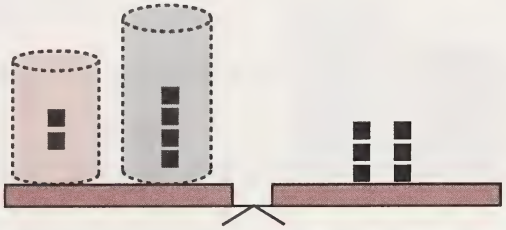


The result is 6 on each side.

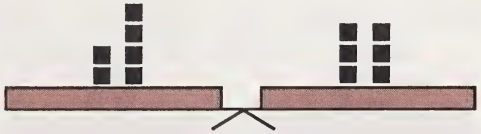
The scale is balanced.

So $a = 1$ and $b = 5$.

Next verify the second solution, replace the cylinder labelled a with 2 and the cylinder labelled b with 4.



Then simplify both sides.



The result is 6 on each side.

The scale is balanced.

So $a = 2$ and $b = 4$.

Can you find other whole number solutions? What are they?

a	b
0	6
3	3
4	2
5	1
6	0

Note

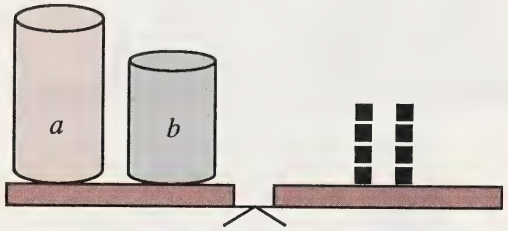
Equations with more than one variable have more than one solution.

If you know how the variables are related, then it is possible to find **one** solution for an equation with two unknowns.

Example

Solve $a + b = 8$ if a is 2 more than b.

First model the equation.



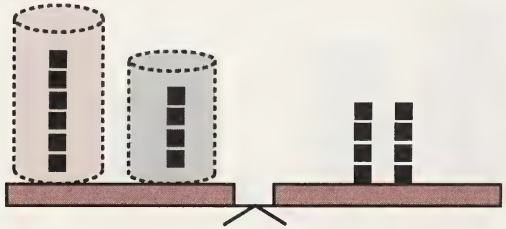
Then think of combinations in which a is 2 more than b.

a	b
6	4
5	3
4	2
3	1
2	0

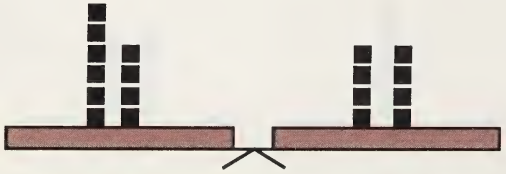
Trial 1

Does $a = 6$ and $b = 4$ make the equation true?

Replace the cylinder labelled a with 6 units and the cylinder labelled b with 4 units and check.



Simplify each side.



The result is 10 on the left-hand side and 8 on the right-hand side.

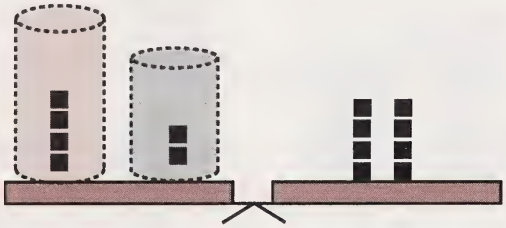
The scale is not balanced.

So $a = 6$ and $b = 4$ is **not** a solution.

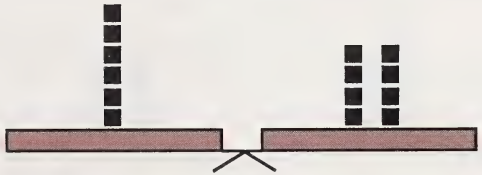
Trial 2

Does $a = 4$ and $b = 2$?

Replace the cylinder labelled a with 4 units and the cylinder labelled b with 2 units.



Simplify each side.



The result is 6 units on LHS and 8 units on RHS.

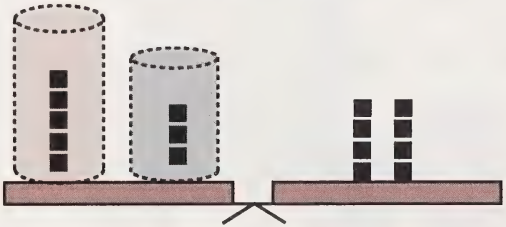
The scale is not balanced.

So $a = 4$ and $b = 2$ is **not** a solution.

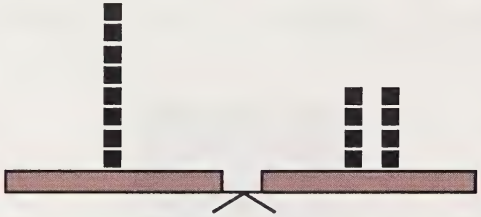
Trial 3

Does $a = 5$ and $b = 3$?

Replace the cylinder labelled a with 5 units and the cylinder labelled b with 3 units.



Simplify each side.



The result is 8 units on each side.

The scale is balanced.

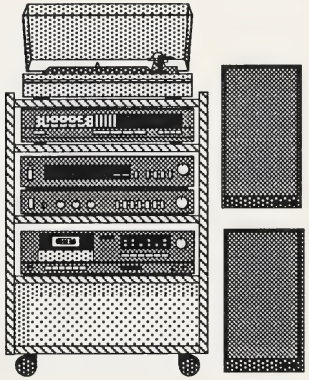
So $a = 5$ and $b = 3$ is a solution.

Concluding Activities

Space for Your Work

Write equations to describe these situations.

1. The stereo costs the money Stacey has saved plus $25.



2. Tuan's mass decreased by 10 kg equal Ruth's mass.

3. The sum of three whole numbers is 24.

4. The length of the house is 3 more than twice the width.

Space for Your Work

Use equations scale, "and" and "take away" cards, cylinders and base 10 blocks in the following.

5. a. Model $a + 2 = c$.

b. Find 2 solutions.

Space for Your Work

c. Verify the solutions.

Space for Your Work

6. a. Model $x + y + 3 = 8$.

b. Find 2 solutions.

Space for Your Work

c. Verify the solutions.

Space for Your Work

7. a. Model $2a + b = 7$.

b. Find 2 solutions.

Space for Your Work

c. Verify the solution.

Space for Your Work

8. a. Model $a + b = 16$.

b. Solve if a is 3 times b.

Space for Your Work

c. Verify the solution.

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will solve equations and verify the solutions without using learning aids.

Working Together

In the previous section you solved equations using learning aids. In this section you will solve the equations by guessing and checking. You will verify the solutions by substitution.

Example 1

Solve $y + 5 = 12$.

Think of a number that makes the equation true.

5 added to what number gives 12?

$\boxed{7} + 5 = 12$

So $y = 7$.

Check

LHS	RHS
$y + 5$	12
$= 7 + 5$	
$= 12$	

LHS = RHS

So $y = 7$.

Example 2

Solve $5k = 55$.

Think of a number that makes the equation true.

5 times what number is 55?

$5 \times \boxed{11} = 55$

So $k = 11$.

Check

LHS	RHS
$5k$	55
$= 5 \times 11$	
$= 55$	

LHS = RHS

So $k = 11$.

Introductory Activities

Space for Your Work

Solve each of the following by guessing and checking. Then verify each solution.

1. $12 + a = 15$

2. $20n = 200$

Space for Your Work

3. $b - 5 = 7$

4. $3a + 2 = 14$

Space for Your Work

5. $5a - 3 = 7$

See your learning facilitator to check your answers and to receive further instructions.

Practice Activities

Space for Your Work

Write equations for the following. Then solve using guess and check method. Be sure to check the solution.

1. Five times the cost is 24.95. What is the cost?

2. The number of children in the class less three is 35. How many children are there in the class?

Space for Your Work

3. Two times the number of goals plus four is 30. What is the number of goals?

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Write an equation for the following. Then solve using guess and check methods. Verify the solutions.

1. Matt is eight years older than Jon. Together their ages total 29 years. How old are they?

2. The sum of two consecutive whole numbers is 25. What are the whole numbers? (Consecutive numbers are in order.)

Space for Your Work

3. Stu answered 30 questions. He answered four times as many questions correctly as he did incorrectly. How many questions did he answer correctly?

4. A 5.6 m rope is cut into two pieces. One piece is three times as long as the other. What are the length of the pieces?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn to describe relationship between pairs of numbers, in words, by an equation, by ordered pairs, and by a graph.

In this section you will review these words.

- axis (plural axes)
- grid
- graph
- plot
- coordinates
- coordinate plane
- ordered pair
- origin
- cartesian plane

Working Together

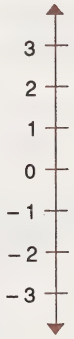
In the Module Introduction you learned Descartes developed the process of graphic algebraic relations.

In this section you will describe relations by graphing, and by other methods.

First, however, you will review number lines and number planes.

Number Lines

As you recall from Modules 2 and 3, number lines can be drawn vertically or horizontally.



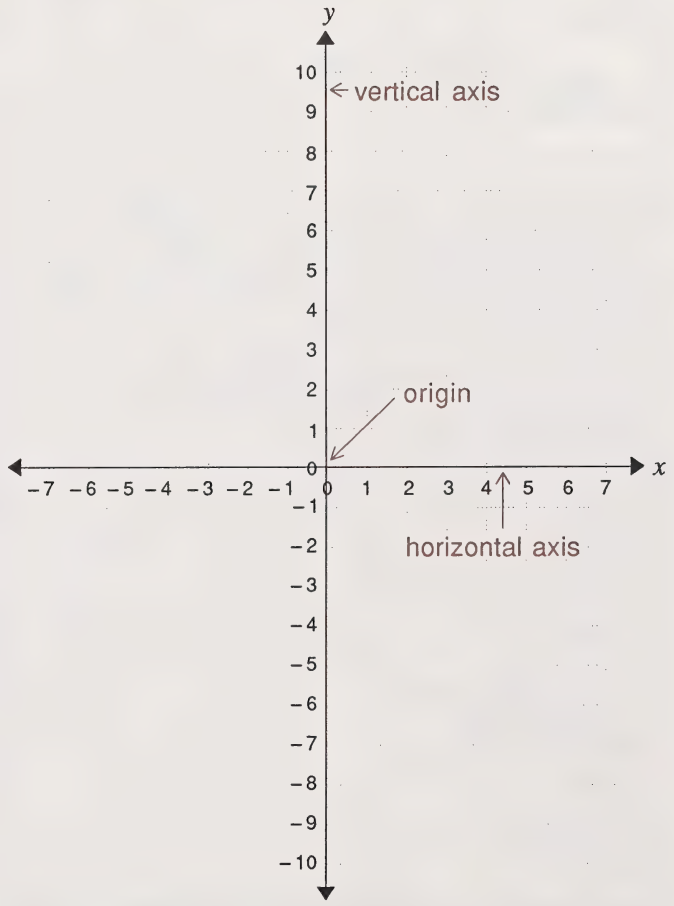
Number Planes

A number plane is formed when two number lines are combined.

The first number line is drawn horizontally and is called the **horizontal axis.**

The second number line is drawn vertically and is called the **vertical axis.**

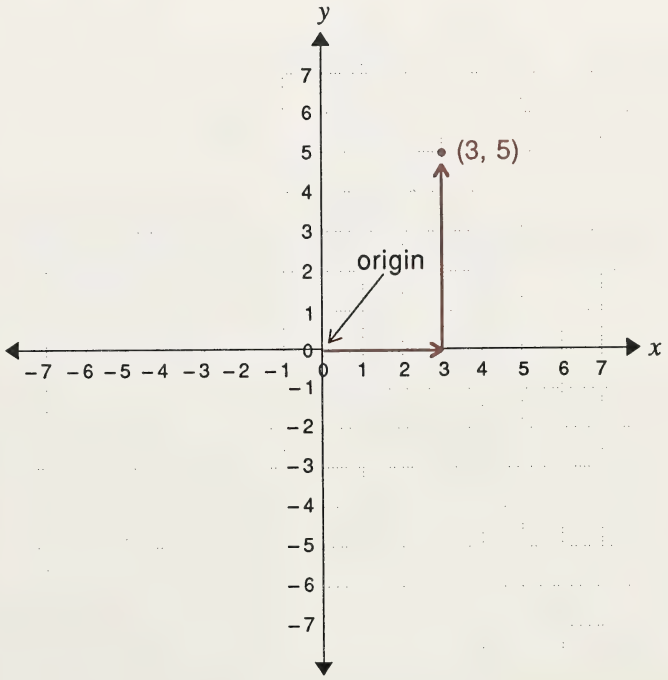
Where the two axes cross is called the **origin.**



Let's review how to plot a point on a grid to represent an ordered pair.

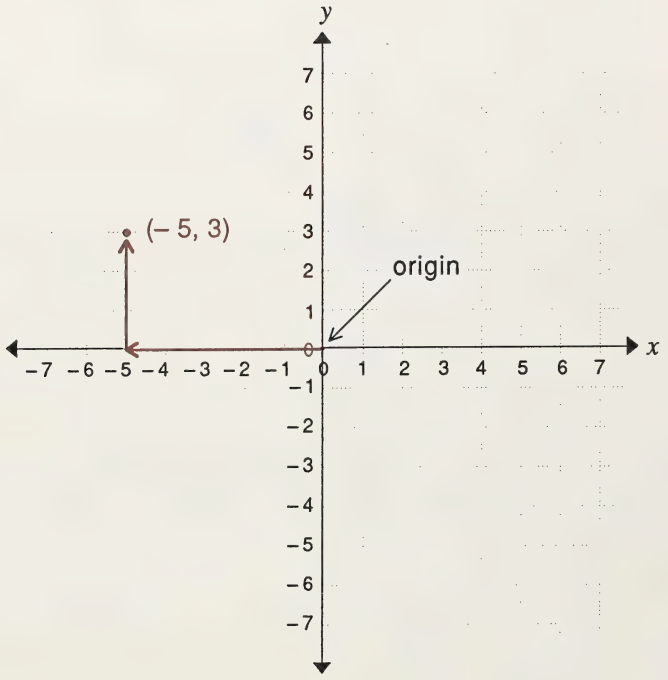
Example 1: Plot (3,5)

Begin at the origin and count right 3 and up 5.



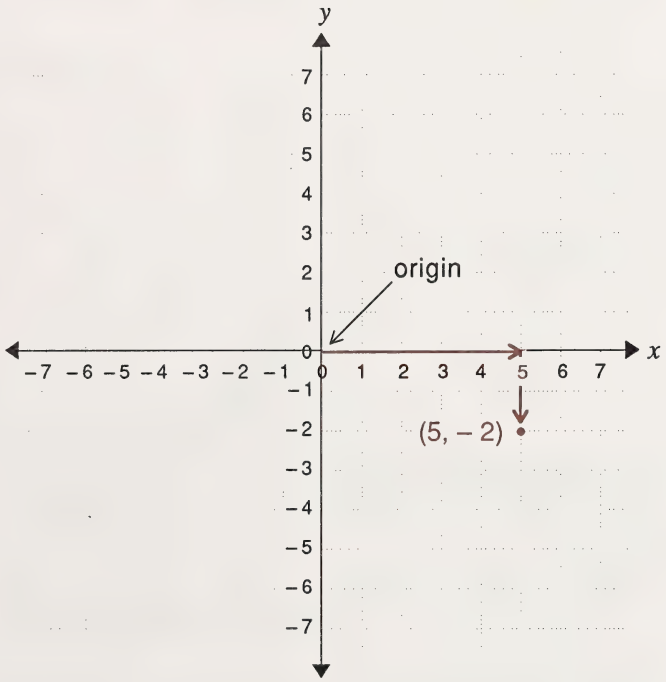
Example 2: Plot (−5,3)

Begin at the origin and count left 5 and up 3.



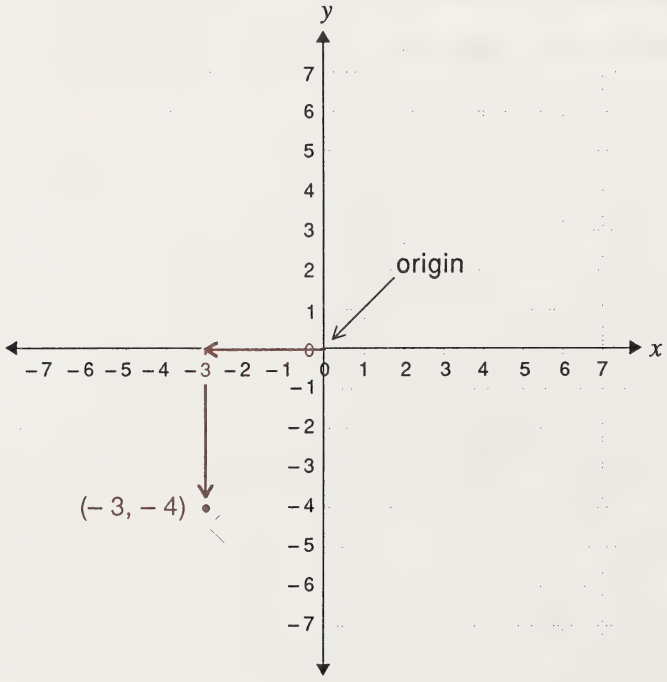
Example 3: Plot $(5, -2)$

Begin at the origin and count right 5 and down 2.



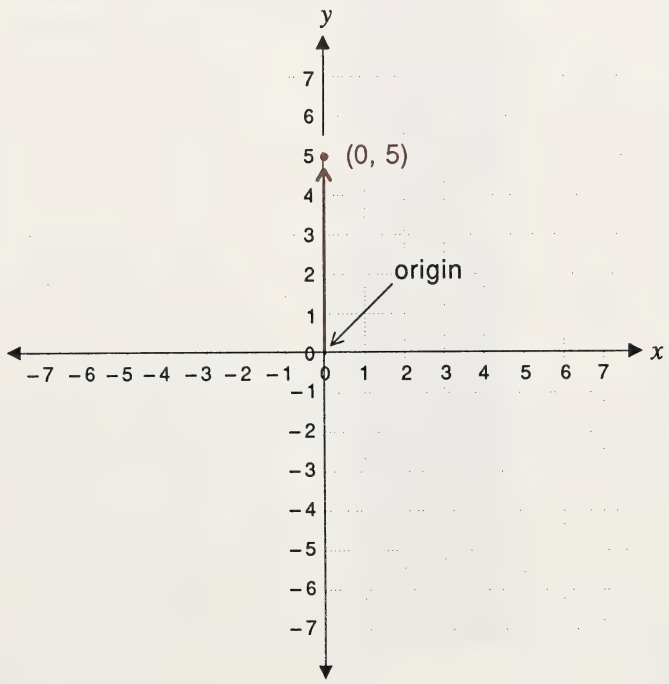
Example 4: Plot $(-3, -4)$

Begin at the origin and count left 3 and down 4.



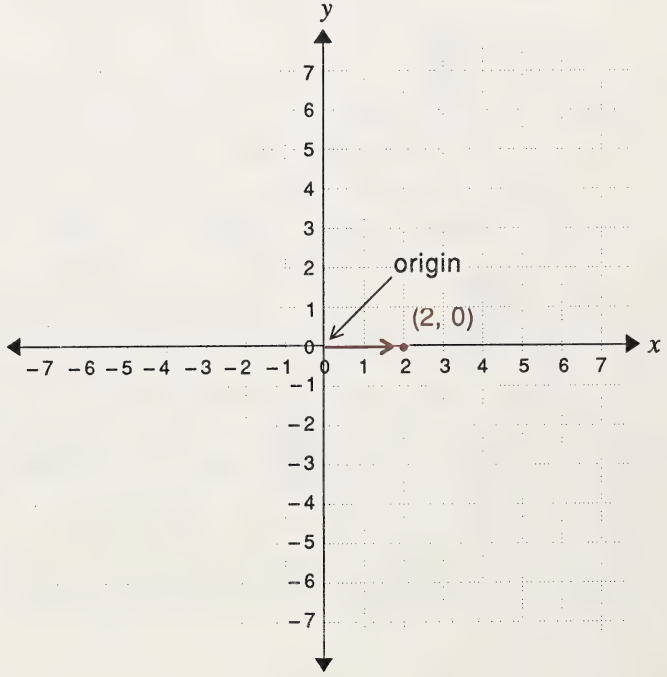
Example 5: Plot (0,5)

Begin at origin. There is no movement right or left. Count up 5.



Example 6: Plot (2,0)

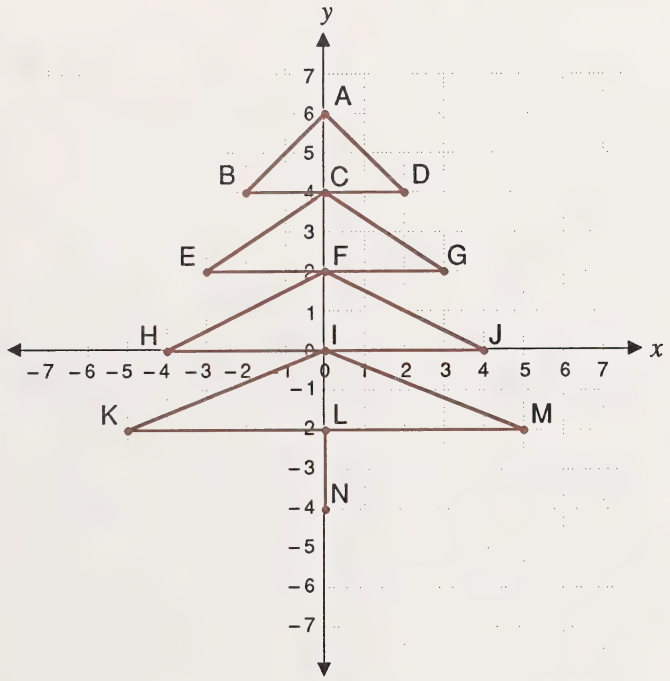
Begin at the origin, count right 2. There is no movement up or down.



Introductory Activities

Space for Your Work

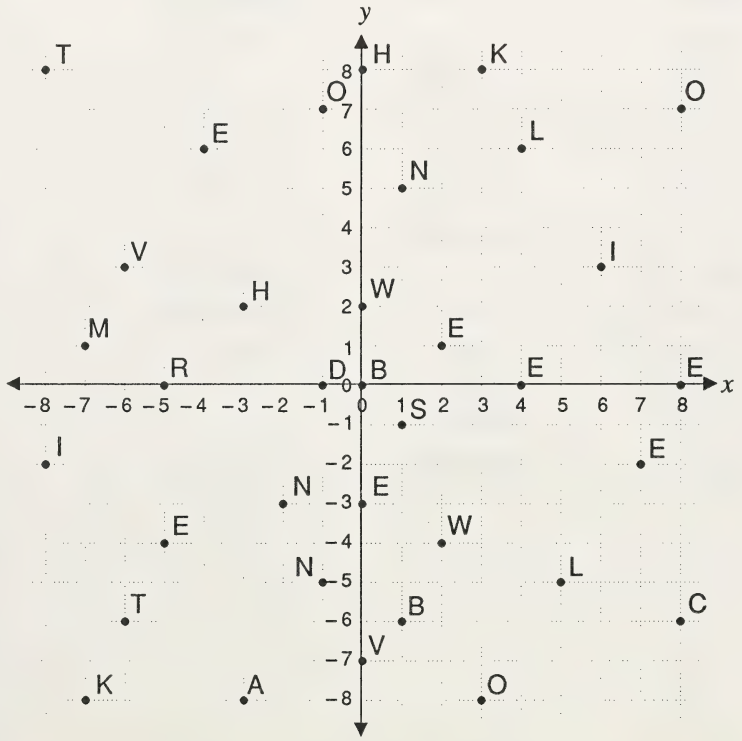
1. Write the coordinates of the points marked.



Point	Coordinates
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	
K	
L	
M	
N	

Space for Your Work

2. Each pair of numbers at the right of the page stands for a point on the coordinates below. Above each pair of numbers, write the letter that appears at that point.[1]



(6,3)		(1,5)	(−4,6)	(−6,3)	(4,0)	(−5,0)

(−7,−8)	(−1,−5)	(7,−2)	(2,−4)		(0,2)	(0,−3)

(0,0)	(8,7)	(−6,−6)	(−3,2)

(5,−5)	(−8,−2)	(0,−7)	(−5,−4)	(−1,0)	(3,−8)	(−2,−3)

(−8,8)	(0,8)	(8,0)		(1,−1)	−3,−8)	(−7,1)	(2,1)

(1,−6)	(4,−6)	(−1,7)	(8,−6)	(3,8)

[1]©1982 Creative Publications, Sunnyvale, California 94086.

3. Plot each ordered pair on the graph. Connect the points in order. A picture will develop[1].

Start
(−6, 6)
(−4, 6)
(−4, 10)
(−6, 10)
(−6, 6)
Lift pencil.

(−3, 5)
(−1, 5)
(−1, 10)
(−3, 10)
(−3, 5)
Lift pencil.

(−6, −4)
(−4, −4)
(−3, −3)
(−3, −1)
(−4, 0)
(−6, 0)
(−7, −1)
(−7, −3)
(−6, −4)
Lift pencil.

(0, 3⅓)
(1⅓, 3⅓)
(1⅓, 2⅔)
(2⅔, 2⅔)
(2⅔, 2)
(5⅓, 2)
(5⅓, 10)
(4, 10)
(4, 11)
(0, 11)
(0, 3⅓)
Lift pencil.

(10, −4)
(12, −4)
(13, −3)
(13, −1)
(12, 0)
(10, 0)
(9, −1)
(9, −3)
(10, −4)
Lift pencil.

(−9, 0)
(−7, 0)
(−7, 1)
(−3, 1)
(−3, 0)
(6, 0)
(6, −2)
(8, −2)

(8, 0)
(9, 0)
(9, 1)
(13, 1)
(13,3)
(11, 3)
(11, 5)
(6, 5)
(6, 11)
(4⅔, 11)
(4⅔, 12)
(−7⅔, 12)
(−7⅔, 11)
(−9, 11)
(−9, 7)
(−8, 7)

(−8, 6)
(−7, 6)
(−7, 5)
(−6, 5)
(−6, 2)
(−8, 2)
(−8, 1)
(−9, 1)
(−9, 0)
Lift pencil.

(7, 5)
(7, 6)
(8, 6)
(8, 5)
Lift pencil.

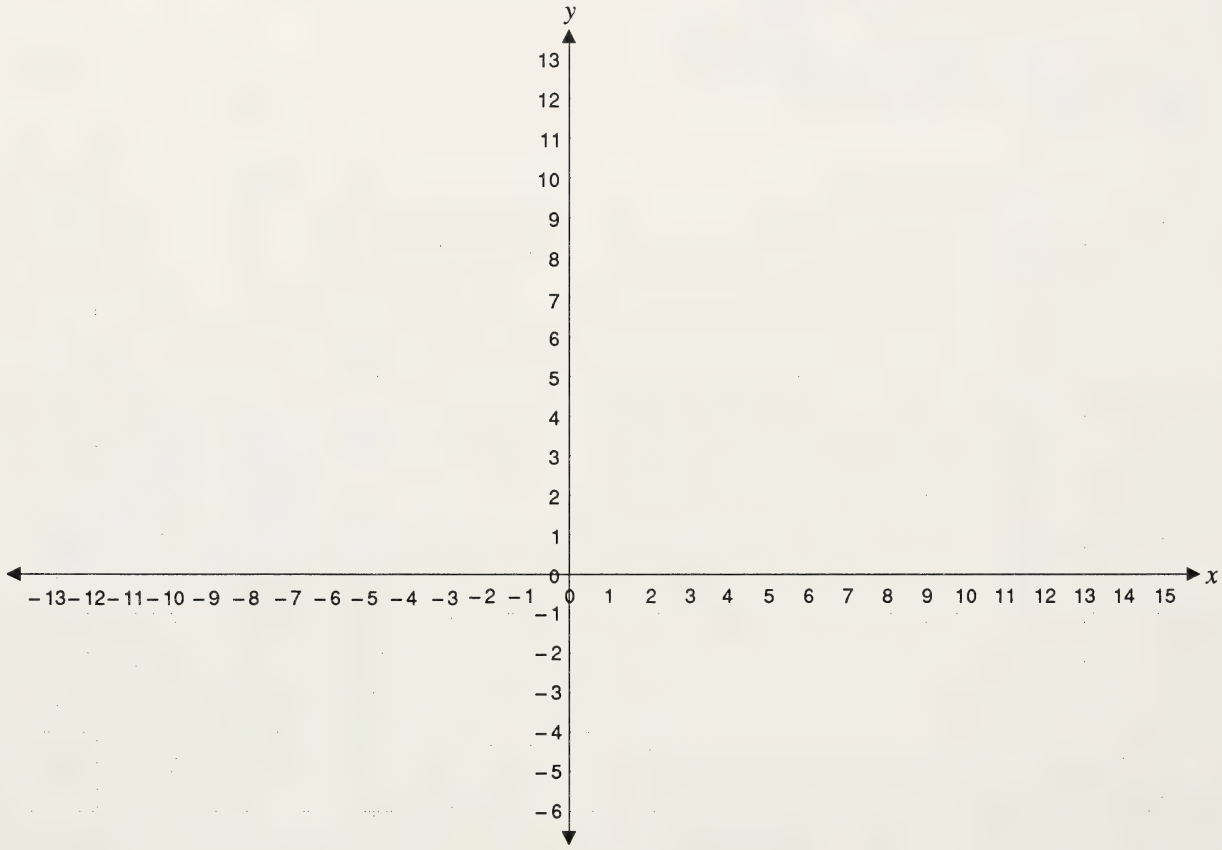
(12, 3)
(12, 4)
(12½, 4)
(12½, 3)
Lift pencil.

(5⅓, 3)
(4, 4)
Lift pencil.

(4⅔, 4⅔)
(3⅓, 3⅓)
Stop

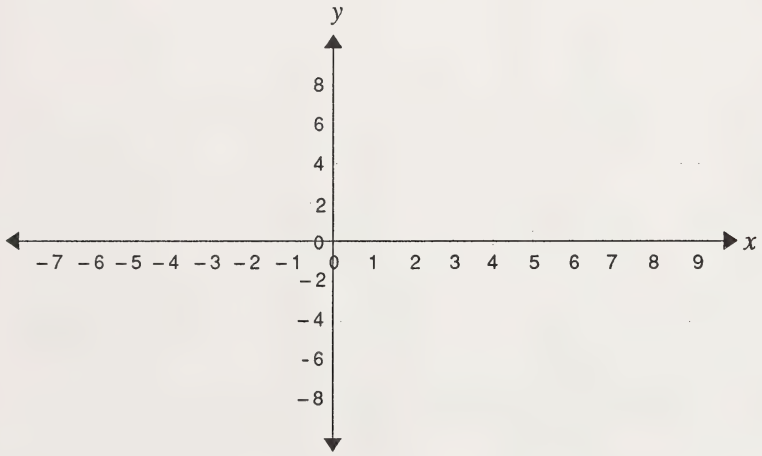
[1]*Arithmetic Teacher,* December, 1985.

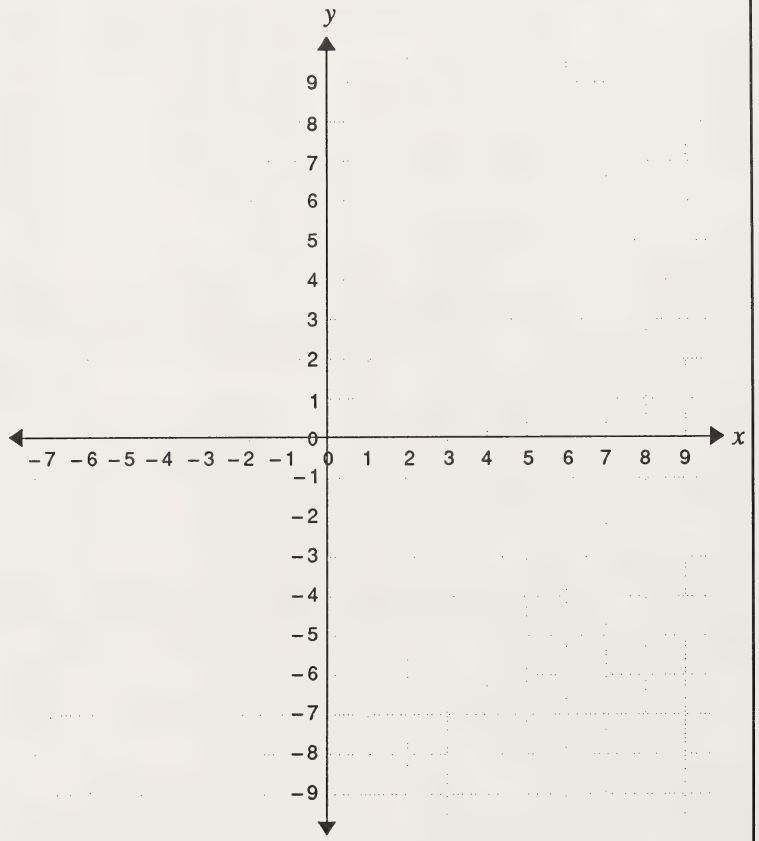
Space for Your Work



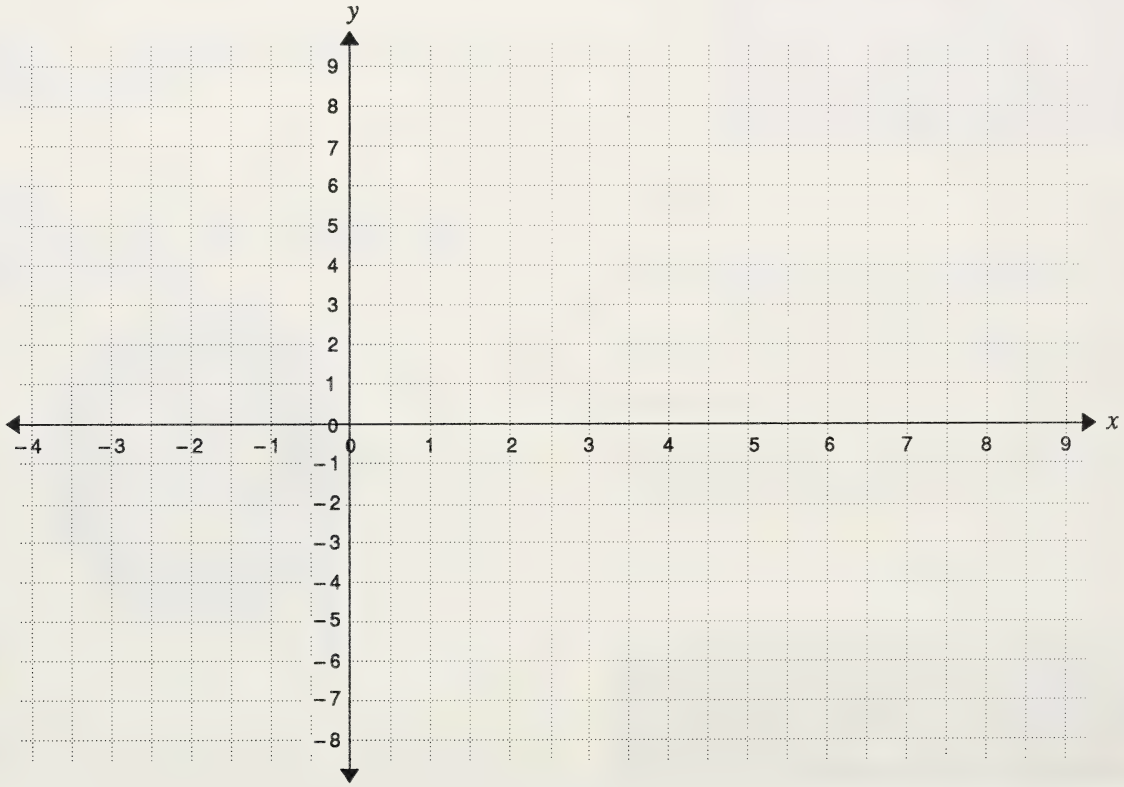
Space for Your Work

4. Plot these points on the 3 number planes provided. Join the points in order. (−4,0), (−3,3), (−2,5), (0,7), (2,8), (3,8), (5,7), (7,5), (8,3), (9,0), (3,0), (3,−6), (2,−8), (0,−7), (0,−6), (1,−7), (2,−7), (2,0), (−4,0).





Space for Your Work



Space for Your Work

5. Why are the pictures in question 5 different?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

In the Introductory Activities you reviewed how to plot points.

Now you will learn to describe relations using several methods including graphing.

Example 1

Norman and Bob are brothers. When Bob was 7, Norman was 9. When Bob was 8, Norman was 10. How are their ages related?

The relationship can be expressed in several ways.

Method 1

The relationship can be shown in the table:

Bob's age (b)	relation	Norman's age (n)
1	1 + 2	3
2	2 + 2	4
3	3 + 2	5
4	4 + 2	6
5	5 + 2	7

Method 2

The relationship can be generalized in words.

Norman's age is 2 years older than Bob's.

Method 3

The relationship can also be described using an equation.

$$n = b + 2$$

Method 4

The relationship between Bob's and Norman's age can be shown by ordered pairs.

Bob's age (b)	Norman's age (n)
1	3
2	4
5	7
6	8
7	9

The ordered pairs are

(1,3), (2,4), (5,7), (6,8), (7,9)

Note

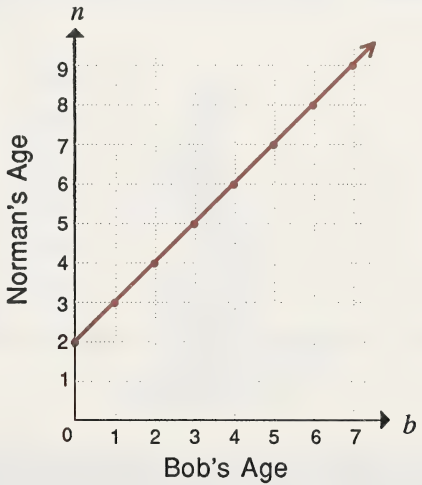
Ordered pairs are always written in parenthesis. A comma separates the ordered pairs.

Method 5

The relationship can be described by number lines.

Method 6

The relationship can be described by a graph.



Example 2

This bank contains only dimes. How is the amount of money in the bank related to the number of dimes?

Solution

If there is 1 dime, there is 10 × 1, or 10 cents.

If there are 2 dimes, there is 10 × 2, or 20 cents.

The value of the dimes in the bank is related to the number of dimes.

The relationship can be described in several ways.

Method 1

The relationship can be shown in the table.

Number of dimes (n)	Relation	Value in cents (v)
1	10 × 1	10
2	10 × 2	20
3	10 × 3	30
4	10 × 4	40
5	10 × 5	50

Method 2

The relationship can be generalized in words.

The value in cents is 10 times the number of dimes.

Method 3

The relationship can also be described using an equation.

$$v = 10n$$

Method 4

The relationship between the number of dimes and value in cents can be described by ordered pairs.

Number of dimes (d)	Value in cents (v)
0	0
1	10
2	20
3	30
4	40
5	50

Ordered pairs are written like this

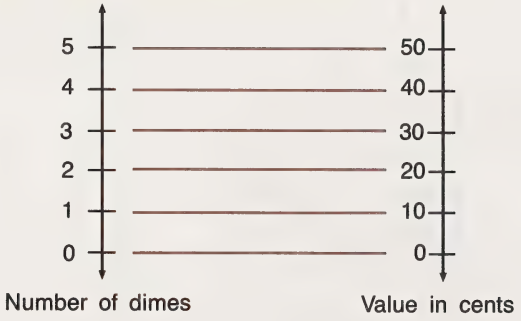
(0, 0), (1, 10), (2, 20), (3, 30), (4, 40), (5, 50)

Note

The ordered pairs are all written in the same order. (The first number is the number of dimes and the second number is the value.)

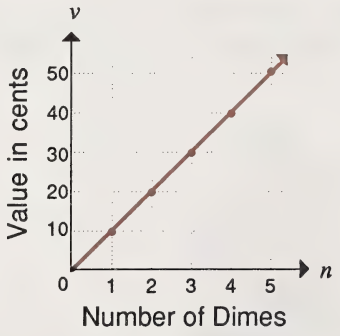
Method 5

The relationship can be described by number lines.



Method 6

The relationship can be described by a graph.



Practice Activities

Space for Your Work

1. Janice likes to go horseback riding. How is the cost related to the riding time?

riding time in hours (t)	relation	cost in dollars (c)
1	$4 + 2 \times 1$	6
2	$4 + 2 \times 2$	8
3	$4 + 2 \times 3$	10
4	$4 + 2 \times 4$	12
5	$4 + 2 \times 5$	14

Describe the relationship several ways.

a. Write words to describe the relation.

b. Write an equation to describe the relation.

Space for Your Work

c. Write the ordered pairs.

d. Describe the relationship using number lines.

Space for Your Work

e. Describe the relationship using a graph.

Space for Your Work

2. How is Rajah's hourly pay related to Nadia's pay?

Nadia's pay (f)	relation	Rajah's pay (g)
5	5 – 1	4
6	6 – 1	5
7	7 – 1	6
8	8 – 1	7
9	9 – 1	8

Describe the relationship several ways.

a. Write words to describe the relation.

b. Write an equation to describe the relation.

c. Write the ordered pairs.

Space for Your Work

d. Use number lines to describe the relation.

Space for Your Work

e. Use a graph to describe the relation.

Space for Your Work

3. How is the length (metres) of the shed related to the width (metres)?

width in metres (w)	relation	length in metres (ℓ)
1	$2 \times 1 + 3$	5
2	$2 \times 2 + 3$	7
3	$2 \times 3 + 3$	9
4	$2 \times 4 + 3$	11
5	$2 \times 6 + 3$	13

Describe the relationship several ways.

a. Describe the relation using words.

b. Describe the relation using an equation.

c. Describe the relation using ordered pairs.

Space for Your Work

d. Describe the relation using number lines.

Space for Your Work

e. Describe the relation using a graph.

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this summary you will review the skills you have learned in this module.

Working Together

Your goal in this module has been to learn skills and words about algebra. To discover how much you have learned, turn to Section 2 and review the list of skills in the "What Lies Ahead" box.

Then correct any errors you made when you did the pretest. You will probably be pleased to discover you have learned a great deal!

MODULE CONCLUSION

What Lies Ahead

The Module Assignment in this Module Conclusion will evaluate the achievement of objectives for this Module.

Working Together

Now that you have studied Module 4 and you have done the required practice, you should be ready for the Module Assignment.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment independently. You may refer to your notes, but do not get help from anyone. Afterwards, submit the assignment for a grade and feedback from your teacher.

APPENDIX

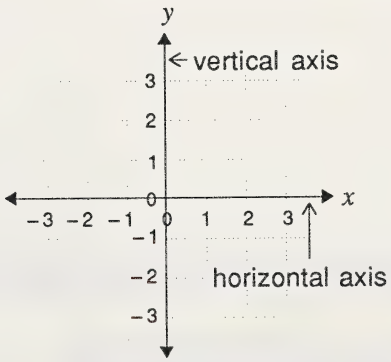
GLOSSARY

Algebra: the branch of mathematics which describes basic arithmetic relations

Algebraic expression: a combination of numerals, variables and/or other mathematical symbols

$3a + 2$

Axis (plural axes): either of the intersecting number lines of a graph



Cartesian plane: a number grid on a plane

Coordinates: the two numbers in an ordered pair that locates a point

Coordinate plane: a number grid on a plane

Equal: identical in value

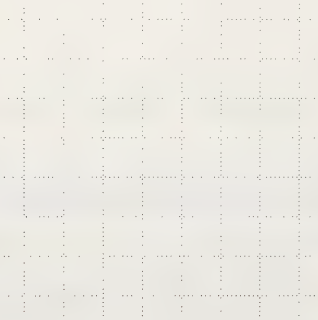
Equation: a number sentence showing the left-hand side and the right-hand side are equal

$7 + 3 = 2 + 8$

Evaluate: to find the value of an expression

Graph: a pictorial device that displays a relationship

Grid: a pattern of lines or dots

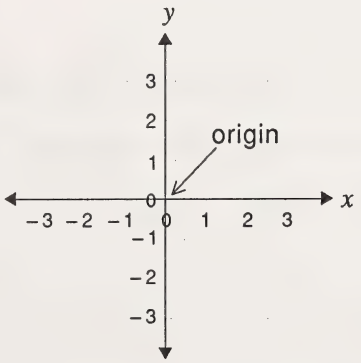


Mathematical expression: a combination of numerals and mathematical symbols

$3 + 2$

Ordered a pair of numbers in which order is important

Origin: the point where the number lines of a graph intersect



Plot: to locate a point on a grid by means of coordinates

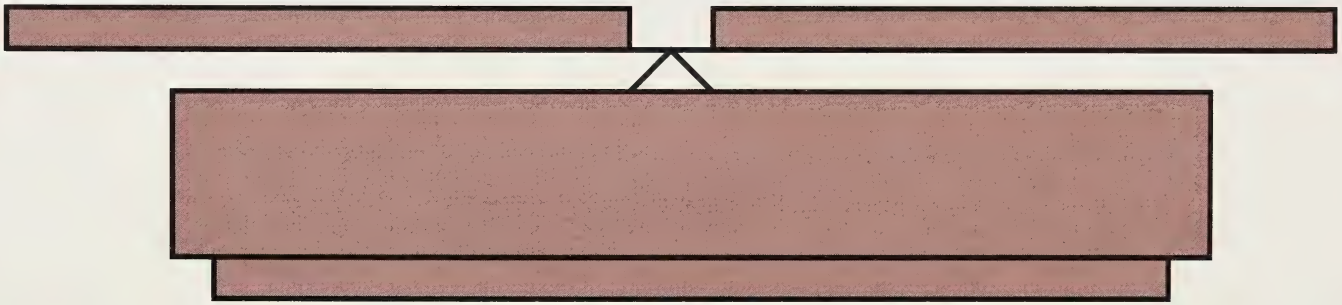
Solve an equation: to find the value of the variable which will make an equation a true statement

Solution: a value of the variable which when used in place of the variable in an equation gives a true statement

Variable: a symbol (usually a letter) used to represent an unspecified or unknown number

Verify: to check the solution

EQUATION SCALE



OPERATION CARDS

and	and	and
take away	take away	take away

N.L.C. - B.N.C.
3 3286 10811482 4

Mathematics 7

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